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ABSTRACT

This study examined factors that influenced the acquisition of pedagogical content knowledge for two groups of prospective secondary mathematics teachers (undergraduate mathematics majors and post-graduate scientists and engineers seeking mathematics certification) in the content domain of functions and graphs. Both groups, enrolled in a one-semester methods course, were assessed on a wide variety of tasks designed to investigate the acquisition of pedagogical content knowledge. Participants also completed a beliefs scale as the course commenced. The primary objective of the study was to document evidence of their subject-matter knowledge; their beliefs about learners, learning mathematics, and mathematics; and how their knowledge and beliefs influenced their instructional practices. The results suggested that subject-matter knowledge and beliefs among both groups were very similar. Results also indicated that the complex endeavor of acquiring pedagogical content knowledge was significantly influenced by the content knowledge and the belief structures that prospective teachers held when entering the methods class. An appendix contains the tasks that participants completed. (Contains 36 references.) (SM)

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"Alternative Pathways to Teaching" - An Investigation of the Factors that Influence the Acquisition of Pedagogical Content Knowledge for Traditional and Non-Traditional Teachers

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Abstract

This study examined the factors that influence the acquisition of pedagogical content knowledge for two groups of prospective secondary mathematics teachers - undergraduate mathematics majors and post-graduate scientists and engineers seeking mathematics certification - in the content domain of functions and graphs. Both groups, enrolled in a one-semester methods course, were assessed on a wide variety of tasks designed to investigate the acquisition of pedagogical content knowledge. The primary objective of this study was to document evidence of their subject-matter knowledge, their beliefs about learners, learning mathematics, and mathematics, and how the knowledge and beliefs influenced their instructional practices. The findings suggest that the complex endeavor of acquiring pedagogical content knowledge is significantly influenced by the content knowledge and the belief structures which prospective teachers hold when entering the methods class.

Conceptual Framework

From the perspective of both teaching and learning, an examination of the factors that influence the acquisition of pedagogical content knowledge for different groups of prospective teachers is an important topic within the conceptual framework that deals with research on teaching (Brophy, 1991; Shulman, 1986). An examination of two groups of prospective teachers with significantly different subject-matter knowledge provides the opportunity to compare and contrast the two groups' belief structures and instructional strategies. In order to describe the relationship between teacher subject-matter knowledge and instructional practices, it is necessary to examine the knowledge teachers have about a particular mathematical topic. The mathematical topic, functions and graphs, was chosen for investigation because it is a major topic in the secondary curriculum and because a significant body of research concerned with students' understanding of functions and graphs has been completed (Leinhardt, Zaslavsky, & Stein, 1991). However, neither an investigation of subject-matter knowledge as a proxy for pedagogical content knowledge nor an investigation that considers beliefs alone can adequately address the complex nature of pedagogical content knowledge. This study linked knowledge and beliefs by examining the influences of knowledge in the content area and beliefs about the learner and mathematics for two important groups - traditional undergraduates and post-graduate scientists and engineers. The following statement from Fennema and Franke (1992) provides a useful perspective with respect to an investigation of teachers' knowledge, beliefs, instructional practices, and the relationship between them.

> The transformation of knowledge in action is understandably complex. Little research is available that explains the relationship between components of knowledge as new knowledge develops in teaching nor is information available regarding the parameters of knowledge being transformed through teacher implementation. Here all aspects of teacher knowledge and beliefs come together and all must be considered to understand the whole. The challenge is to develop methodologies and systemic studies that will provide information to enlighten our thinking in this area. The future lies in understanding the dynamic interaction between components of teacher knowledge and beliefs, the roles they play, and how the roles differ as teachers differ in the knowledge and beliefs they possess. (p.163).

Pedagogical Content Knowledge

Shulman describes pedagogical content knowledge as the knowledge "which goes beyond knowledge of subject-matter per se to the dimension of subject-matter knowledge for teaching" (Shulman. 1986, p.9). He suggests that for a particular subject area such as mathematics, pedagogical content knowledge includes "the most useful representation of those ideas, the most powerful analogies, illustrations, examples, explanations, demonstrations - in a word the ways of representing and formatting the subject that make it comprehensible to others" (Shulman, 1986, p.10).

If Schulman's description is accurate, the question concerning where pedagogical content knowledge comes from and what are its origins remain. As preservice teachers prepare for teaching and for making the transition from students to teachers, it is likely that they draw upon several sources of knowledge and beliefs. First, their own subject-matter knowledge will obviously play a significant part in their efforts to design lessons for students. But, it is reasonable to suppose that subject-matter knowledge will not completely determine the lessons. Teachers' beliefs about how students learn mathematics, their beliefs about mathematics itself, and their knowledge of teaching, in general, are likely to affect how they design and teach lessons. Two teachers who share similar subject-matter profiles may teach differently if they differ along these other dimensions.

Potential Sources of Pedagogical Content Knowledge

Subject-Matter Knowledge

An investigation of prospective teachers' pedagogical content knowledge may be operationalized in the context of a specific content domain. Subject-matter knowledge of mathematics at the secondary level may be examined through a specific topic within the content domain. The mathematical topic functions, graphs, and graphing has received considerable attention of late (Leinhardt, Zaslavsky, & Stein, 1990).

The content domain of function and graphs is important both from the perspective of being a central topic in the secondary curriculum and also from the perspective of serving as a site in which conceptual understanding is dependent on connecting different representational forms. In their comprehensive review of the research literature on "Functions, Graphs, and Graphing," Leinhardt, Zaslavsky, and Stein (1990) suggest that the algebraic and graphical representations of functions are "two very different symbol systems that articulate in such a way as to jointly construct and define the mathematical concept of function" (Leinhardt et al., 1990, p.3).

The current reform movement in mathematics education also emphasizes the importance of forming connections between these two representations of functions. "Students who are able to apply and translate among different representations of the same problem situation or the same mathematical concept will have at once a powerful, flexible set of tools for solving problems and a deeper appreciation of the consistency and beauty of mathematics" (NCTM Curriculum and Evaluation Standards 1989, p.146). Thus, the study of pedagogical content knowledge of prospective secondary teachers within the content domain of functions and graphs provides the opportunity to examine the relationship between knowledge and instructional practices in an important topic in secondary mathematics.

Within the content domain of functions and graphing there have emerged a number of studies which utilize research on students' thinking in this domain to investigate teachers' knowledge about the content and the learner (Even & Markovits, 1993; Even, 1993; Wilson, 1994). Even (1993) found that many of the prospective teachers in her study did not hold a modern conception of function but held naive conceptions that were similar to those of students described in the research literature (Dreyfus & Eisenberg, 1983, 1987; Lovell, 1971; Markovits et al., 1983, 1986; Marnyanskii, 1975; Thomas, 1975; Vinner, 1983; Vinner & Dreyfus, 1989). She suggests that this limited conception of function clearly impacts teachers' "pedagogical decisions - questions they ask, activities they design, students' suggestions they follow - which are based in part on their subject-matter knowledge" (Even, 1993, p.113). Even and Markovits suggest that within the formal preparation of teachers there has been little emphasis on the understanding of students' ways of thinking related to specific topics and the development of appropriate ways of responding to students' questions, remarks, and ideas. Thus, their findings that teachers experience difficulty utilizing knowledge of the learner in other than procedural ways, do not seem surprising (Even & Markovits, 1993). The results of the studies that investigate teachers' knowledge about both the content domain and students' understanding of functions and graphs suggest that this knowledge is an essential source of pedagogical content knowledge.

Knowledge of Students' Learning

In addition to viewing pedagogical content knowledge as the ability to represent and formulate subject-matter knowledge so that it is comprehensible to others, some researchers (Even & Markovits, 1993) also view pedagogical content knowledge as an understanding of what makes the learning of specific topics easy or difficult and the knowledge of conceptions and preconceptions that students of different ages and backgrounds bring with them to learning a specific topic. If this is the case, knowledge about students' learning as well as beliefs about learning will make a difference in their pedagogical content knowledge.

Research (Carpenter, Fennema, Peterson, Chiang, & Loef, 1989) suggests that, in some domains, teachers can use domain specific knowledge of children's learning to make specific instructional decisions, and thus, impact learning. The CGI studies provide evidence that teachers can attend to individual students when they have appropriate and well-organized knowledge. The specific, well-organized knowledge which the CGI teachers had access to enabled them to understand children's thinking in ways they had been unable to before.

These results emphasize the importance of teachers' knowledge about the learner in general (as a constructor of knowledge) and specifically (in a particular content domain) and how this knowledge has the potential to impact instruction and subsequently learning. However, as Hiebert and Wearne (1991) point out, there does not exist a robust integrated set of knowledge available about most content areas within the mathematics curriculum. Thus, this powerful knowledge about students' understanding within a particular content domain may not be either available or accessible to teachers.

Beliefs about Learners and Learning Mathematics

The close association of cognitive psychology with research in the teaching and learning of mathematics has contributed extensively to the view of the learner in mathematics. Students are no longer viewed as "blank slates" but as active constructors of their own knowledge. The basic tenet that learners are active in structuring and inventing knowledge has important implications for teaching mathematics. Instruction cannot be viewed as the simple presentation, however carefully dome, of the knowledge and skills to be acquired. Instruction must focus on the means to facilitate construction of mathematical knowledge (built on existing knowledge) through providing classroom settings in which students (learners) explore relationships, use those relationships as tools to solve problems, and communicate those findings with each other and the teacher.

For the purposes of this study, these contrasting beliefs about the learner and learning mathematics have important implications. Because these two views are so different, it is reasonable to suppose that two teachers with similar subject-matter knowledge, but different beliefs about the learner and learning mathematics would engage in different instructional practices. Thus, the prospective teachers' beliefs about the learner and learning mathematics matter as we consider evidence of how potential sources of pedagogical content knowledge are reflected in the early simulations of instructional practice.

Beliefs About Mathematics

Teachers' beliefs about mathematics may impact the nature of their pedagogical content knowledge. Beliefs about mathematics are particularly important because they may affect the form in which the concepts and skills are conveyed. Teachers who believe that mathematics is to be discovered and constructed by the students may design a lesson in which the students explore quantitative relationships initially by means of a table, then use a graphing calculator, and finally determine an algebraic representation of the solution. Teachers who believe that mathematics is a fixed body of knowledge to be transmitted may design quite a different lesson on the same topic.

Cooney (1985) has argued that substantive changes in the teaching of mathematics such as those proposed by the current reform documents (NCTM Curriculum and Evaluation Standards for School Mathematics, 1989) will be slow in coming and difficult to achieve because of the basic beliefs teachers hold about mathematics. He notes that the most prevalent verb used by preservice teachers to describe their teaching is *present*. This suggests the existence of a fixed body of knowledge to be transmitted to the learners. This is significant because teachers' view of how teaching should take place in the classroom is strongly influenced by the teachers' understanding of the nature of mathematics (Hersh, 1986).

Even & Markovits (in press) describe several dimensions of analysis which relate to teachers' beliefs about the nature of mathematics. They noted that the majority of experienced teachers' responses to vignettes of students' misconceptions were teacher-centered indicating a transfer of knowledge by direct telling, and few were rich in conceptual meaning with the majority comprising essentially procedural explanations of the problem. Thus, the instructional strategies utilized by these teachers seem to indicate the belief that mathematics is a fixed body of knowledge that must be transferred from teacher to student. These kinds of results provide evidence of teachers' beliefs about the nature of mathematics and suggest the strong influence of beliefs about mathematics on pedagogical content knowledge.

Activities That Might Stimulate the Development of Pedagogical Content Knowledge

The importance of pedagogical content knowledge and its impact on quality instruction is evident within the research on teaching. Pedagogical content knowledge may be the product of integrating knowledge (Ball, 1988). "When teachers represent mathematics they are influenced by what they know across different domains of knowledge: mathematics, learning, learners, and context" (Even & Markovits, 1993, p.25). Developing pedagogical content knowledge is a gradual and complex process. One way to better understand its development is to focus on those sites where its construction may occur. For prospective teachers, these sites are likely to include the development or construction of explanations which include not only explanations but also analogies, representations, examples, and demonstrations, the planning of lessons, the teaching or simulation of teaching, and reflection on teaching.

Methodology

The subjects chosen for this study consist of 10 teacher credential students from the Lawrence Livermore National Laboratory (LLNL) enrolled in a special program designed for them by San Jose State University and 10 undergraduate secondary mathematics students enrolled in a Secondary Methods course at the University of Delaware. Tasks were designed to assess different aspects of pedagogical content knowledge within the discipline of mathematics and the content area of functions and graphs. **Subject-matter knowledge** was assessed through an analysis of their academic transcripts with respect to GPA and number of hours of course work in mathematics, statistics, or computer science. In addition, all of the subjects were given a written assessment of subject-matter knowledge of functions and graphs. **Beliefs** were assessed through a 32-item Likert scale which specifically addressed beliefs about the learner, learning mathematics, and mathematics itself. **Instructional practices** were assessed through a vignettes task in which all of the subjects were asked to respond to student misconceptions and/or questions concerning major topics about functions and graphs - definition, notation and evaluation, composition, and inverse functions.

Subjects

The subjects chosen for this study consist of two groups of prospective secondary mathematics students - undergraduate mathematics majors and post-graduate scientists and engineers. The undergraduates (undergrads) were all full time mathematics majors seeking either a BA or BS with a teaching credential. All of the undergraduates were in their early 20's. The students from the Lawrence Livermore/San Jose State teacher training program were all experienced professional research scientists and mathematicians working full time at the laboratory. Thus, they were part-time students seeking a mathematics teaching credential and represented a range of ages from their 30's to their 50's. All of the "lab students" possessed undergraduate degrees in mathematics, computer science, engineering, or physics with 75% possessing a Master's Degree and 25% possessing a Ph.D.

Setting

The undergraduate students were enrolled in a traditional one-semester secondary methods course which represents the capstone of their formal coursework prior to student teaching. All of these students had taken the required education courses and most had fulfilled their mathematics requirements. Thus, they were familiar with educational theories of learning and some of the reform documents in mathematics education such as the Curriculum and Evaluation Standards for School Mathematics. In contrast, the "lab students" were enrolled in a special part-time off-campus program designed for them. The "methods course" in which the data was collected represented the initial experience in the teacher preparation program. Unless they had read educational theory or some of the documents related to reform in mathematics and science education on their own, these concepts would be new to them.

Tasks

The tasks that were utilized for the purposes of this study represent only a portion of the data that was collected for each group of prospective teachers. However, these tasks provide the means to link knowledge and beliefs to potential instructional practices for two important groups by examining the influences of knowledge in the content area and beliefs about the learner and mathematics on their potential instructional practices.

As each course commenced, both groups were asked to complete a Beliefs Scale designed to assess their beliefs about the learner, learning mathematics, and mathematics itself. This scale, designed by one of the investigators, is a Likert scale based on views relevant to the teaching and learning of mathematics. They are listed below.

1. Beliefs about the learner (questions 1-8).
2. Beliefs about learning mathematics (questions 9-20).

    a. Representations
    b. Knowledge structures
    c. Connections between representations

3. Beliefs about mathematics (questions 21-32).
    a. Problem-solving
    b. Mathematizing
    c. Mathematical argument

In the scale. 8 questions were designed to assess the view of the learner (4 positively stated and 4 negatively stated). Twelve (12) questions were designed for both of the categories - view of learning mathematics and view of mathematics - 4 in each sub-category of which 2 were positively weighted and 2 were negatively weighted. Scores on the Beliefs Scale provided the opportunity to place each teacher along a continuum. For example, in the category Beliefs about the Learner, beliefs about the learner could range from the view that students learn by remembering what they are taught to the view that students construct meaning as they learn mathematics. These same types of contrasting beliefs may also be assessed for each of the other categories. Thus, the results from the Beliefs Scale provide the opportunity to determine the prospective teachers' strongly held beliefs for each of these categories and to assess how these beliefs are reflected in the early simulations of instructional practice.

Research on mathematics teachers' beliefs (Cooney, 1985; Thompson, 1984; Putnam, Lampert, & Peterson, 1990) provided the theoretical perspectives that guided the development of the Beliefs Scale. While there are certainly other categories of beliefs which teachers hold, these three seem to capture the most critical beliefs relative to pedagogical content knowledge. Several public policy and curriculum recommendation documents (Everybody Counts, 1989; Curriculum and Evaluation Standards for School Mathematics, 1989; A Call for Change: Recommendations for the Mathematical Preparation of Teachers of Mathematics, 1991) were used to design the types of questions that would reflect contrasting beliefs. The Beliefs Scale, with the items ordered by views relevant to the teaching and learning of mathematics, with all of the positively-weighted items listed first and all of the negatively weighted items listed second for each category, is provided in Appendix A. A scale in which the items were randomly arranged was administered to the subjects.

The first task (I), found in Table A.1. in Appendix A, a written assessment of subject-matter knowledge in the content domain of functions and graphs, consisted of 15 multiple choice questions which may be categorized along the following three dimensions.

1. Interpretation or Construction
2. Initial Presentation
    a. Algebraic
    b. Graphical
3. Topic related to Functions and Graphs
    a. Definition
    b. Notation and Evaluation
    c. Composition
    d. Inverse Function

The following table indicates how the questions were distributed and which questions represented the three dimensions described above.

Table 1. Dimensions Represented by Task I

Topic Dimension	Definition	Notation & Evaluation	Composition	Inverse Functions
Interpretation	Questions 1,2	Questions 4,15	Questions 8	Questions 9
Construction	none	6, 7	12, 13	10
Initial Presentation Algebraic	3	5, 6, 15	8, 11, 13	10
Initial Presentation Graphical	2, 4	4, 7, 14	12	9

In coordinating all of the different tasks that would contribute to the emerging profile of pedagogical content knowledge for each prospective teacher, it is important to use a written assessment of subject-matter knowledge prior to the task which asked them to react to students' misconceptions or plan and design instruction. This ordering of the tasks accomplished two objectives for the study: 1) it provided a means to categorize the subjects in order to choose cases for further analysis; and 2) it provided some initial information which could be used to report both consistencies and inconsistencies in subject-matter knowledge revealed in the instructional activities.

The second task (II) found in Table A.2 in Appendix A, was an assessment of prospective teachers' understanding of student conceptions and misconceptions of functions and graphs. In this task the subjects were presented with five scenarios or vignettes describing student comments and were asked to describe how they would respond to the students' in the situation. The content of each vignette represented a combination of the four topics related to the function concept described previously: definition; notation and evaluation; composition of functions; and inverse functions. The first vignette was set in the context of a typical student misconception described in the literature concerning functions and graphs - picture-as-graph misconception.

The first vignette presented a function in which the graph does provide a correct image of the motion of the projectile. The subjects were asked to respond to the students' comment and offer (perhaps) some further explanation. In the research on learning that considers the concept of functions, it has been suggested that students can deal with functions in a point-wise manner but experience difficulty examining the behavior of a function over an interval, in a global way, or as an entity (Bell & Janvier, 1981; Even, 1989; Janvier, 1978; Marnyanskii, 1975; Monk, 1988). These studies also focus on the picture-as-graph misconception in which a student assumes that the graph of the function (position vs. time or velocity vs. time) is represented by a sketch of the path of the projectile, and suggest its persistence in students' thinking. Including this vignette provided the opportunity to assess prospective teachers' subject-matter knowledge of this potentially difficult concept as well as examining their knowledge of students' conceptions.

The second vignette presented a qualitative graph of a function. The subjects were asked to respond to the students' comment that suggests the misconception that functions must be linear. Thus, the vignette addressed a typical misconception reported in the research literature as well as providing another opportunity to assess the subjects' knowledge of the definition of function.

The third vignette presented an evaluation problem in which the student was asked to construct f(x+1) when they were given the equation for f(x). The subjects were asked to evaluate the incorrect solutions in terms of determining the source of the errors and suggesting a response to all of these solutions. This vignette is closely related to the research which examines students' concept image of function as well as representing a major topic within the content domain (evaluation of functions).

The fourth vignette presented a function obtained by composition and asked the subjects to respond to student comments about the nature of two functions f(x) and g(x) where h(x) = f[g(x)]. This vignette is also closely related to the nature of the student's concept image and the way they use symbols to express operations with functions. The subjects were asked to evaluate the student comments and suggest responses to clear up the confusion. This vignette also provided the opportunity to examine subject-matter knowledge and assess prospective teachers' responses to a genuine student dilemma.

The fifth vignette presented a problem in which students were asked to determine the inverse of a given function. The students' comments basically reflect confusion with respect to order of operations and how symbols are used to express operations with functions.

The analysis of the prospective teachers' explanations and responses provided a measure of subject-matter knowledge that may be compared and contrasted with the research on students' concept image of function (Vinner, 1983). The analysis of this task also serves as an important link to those tasks that specifically address pedagogical content knowledge. If teachers hold a limited and somewhat impoverished concept image of functions and graphs, they may have few tools with which to construct the instructional activities that may lead to the development of pedagogical content knowledge. They may also have few resources with which to respond to genuine students' dilemmas, build links between symbol systems, and create authentic contexts for the investigation of functions. Thus, Task II was analyzed with respect to documenting the teachers' responses to students' conceptions and misconceptions and suggestions for facilitating student understanding. In particular, the analysis of this task focused on the following three dimensions - teacher level of discourse/communication and cognition, student level of discourse/ communication and cognition, and ways of "sense making"/ the notion of mathematical authority. The following table describes the dimensions of the Teacher-Student-Mathematical Justification Triad: A Density Distribution of Classroom Interactions. Each of the subjects was assigned an ordered triplet that represented their responses to each of the five vignettes. For each of the vignettes, the distribution of responses, both within and between the two groups of prospective teachers, provide a quantitative measure of the responses. Across all of the vignettes, these measures provide a means to determine whether the responses were mainly teacher-directed or student-centered and what notion of mathematical authority was used.

Table 2: A Density Distribution of Classroom Interactions

X-Axis: Teacher Level of Discourse and Cognition

1. **Passive**

   Observing: Receiving knowledge - Teacher observes students.

2. **Active**

   Giving explanations or information: Sharing knowledge - teacher provides mathematically sound explanation.

3. **Investigative**

Suggesting Inquiry Activities: Reasoning about constructing knowledge - teacher orchestrates/suggests alternative pathways to solve the problem.

Y-Axis: Student Level of Discourse and Cognition

1. **Passive**

Observing: Receiving knowledge - Student observes teacher's explanation.

2. **Active**

Giving explanations or information: sharing knowledge - teacher asks students to explain their procedure or logic to the class for feedback.

3. **Investigative**

Suggesting Inquiry Activities: Reasoning about constructing knowledge - genuine invitation for students to engage in mathematical inquiry.

Z-Axis: Ways of "Sense-Making": The Notion of Mathematical Authority

1. **Enactive**: Related to real-world contextual situations - connected to concrete experiences.
2. **Iconic**: Experimental investigations - numerical substitutions, utilization of technological tools, and the use of counter examples or contradictions.
3. **Symbolic**: Mathematical Formalism - review or establish formal definitions, symbolic representations, and algorithms or procedures.

Results

Overview

The analysis of the academic transcripts for the two groups of prospective teachers revealed expected differences between the two groups. The "undergrads" had an average undergraduate GPA of 3.0 while the "lab" students had an average undergraduate GPA of 3.3. While the difference of three tenths might not be significant when one examines groups of students from two institutions, the 15-20 year difference in when these GPA's were earned undoubtedly is significant. All of the "undergrads" were mathematics majors while the "lab" students' majors were from mathematics, engineering, physics, and computer science. In addition, seven of the "lab" students have a Masters degree and three of these students have a Ph.D. in subjects such as computer science, mathematics, engineering, and physics. These students all have many years experience working in research, many have published a number of research papers, and several have patents.

Subject-Matter Knowledge: Task I

The analysis of Task I for both groups of prospective teachers indicated no differences between the groups with respect to the number of questions answered correctly. The following table indicates those results.

Table 2. Written Assessment of Subject-Matter Knowledge

Category Group	Weak (0-10)	Intermediate (11-13)	Strong (14-15)
Undergraduates	2	4	4
"Lab" Students	2	4	4

In terms of which questions were answered incorrectly, differences in their subject-matter knowledge and experience with functions and graphs were revealed. The following table indicates those questions that were most frequently answered incorrectly and how each group performed on them.

Table 3. Most Frequently Missed Questions on Task I

Question	Undergraduates	Lab Students	Description
#1	1	4	Definition of function - interpretation - arrow diagram, graph, & table
#2	5	2	Definition of function - graphical
#4	4	4	Function evaluation - graphical
#13	4	6	Composition - trigonometric - graphical
#14	4	0	Function evaluation - graphical

Each group exhibited certain strengths and weaknesses with respect to Task I. The "lab" students seemed to have the most difficulty with questions #1, #4, and #13. On question #1 the subjects were asked to determine which of three relations displayed as an arrow diagram, a graph, and a table described functions. Their difficulties could certainly be attributed to a lack of familiarity with relations and functions displayed by arrow diagrams. On question #4 the subjects were asked to determine the truth of several statements relating to the graph of a piece-wise function. The students had to consider whether $f[f(3.5)] > 0$, whether the function is one-to-one, and the range of the function. Thus, their difficulties could be attributed to a lack of understanding in several areas. On question #13 the subjects were asked to construct the graph of $h(x)$ where $h(x) = f[g(x)]$ and $f(x) = 2x$ and $g(x) = \sin(x)$. They were given four choices for the graph of $h(x)$. Their difficulties on this question could be attributed to difficulties with function composition and interpretation of the graphical representation of the function.

The undergraduates seemed to experience the most difficulty with questions #2 and #14 although they also had trouble with question #13. On question #2 the subjects were asked to examine five graphs and determine which represented a function. This result is significant since examining graphs and utilizing the Vertical Line Test as an instantiation of the definition of function is typically routine for even pre-calculus students. For 50% of the undergraduates to find this question challenging suggests that they were quite unfamiliar with functions when this task was administered. On question #14 the subjects were asked to examine the graph of a polynomial function and determine whether $f(0) > f(-2)$ and $f(-3) < 0$ are true statements for this function. As with the other question, these difficulties are also related to the subjects' inability to interpret both the graphical representation of the function and the notation. Similarly, both questions #4 and #13 were also incorrectly answered by 40% of the undergraduates. The common feature of all of these problems seems to be the graphical representation of the function. These findings suggest that the undergraduates experienced many of the same difficulties utilizing the graphical representation to reason about functions as do typical pre-calculus students.

In terms of subject-matter knowledge, one possible hypothesis is that the "lab" students might possess a deep and sustained conceptual understanding of mathematics in general. This hypothesis is plausible because of the types and variety of majors, their advanced degrees, and the nature of the work in which they engage - scientific research and development. Although lack of interaction with the specific kinds of items on Task I could contribute to errors, those errors would not reflect true misconceptions. The evidence from Task I seems to support this hypothesis. Of the three questions answered incorrectly by 40% to 60% of the "lab" students, those questions all reflect specific knowledge of function representation and notation related to definition, evaluation and composition. Given that Task I was administered in their initial experience in the teacher training program, these errors do not seem significant. Their responses to Task II will provide further evidence of their subject-matter knowledge and confirm or refute this hypothesis.

With respect to the undergraduates, one would expect them to possess greater familiarity with function representation and notation because their learning and tutoring experiences are fairly recent and contiguous in time. One would also expect their mathematical knowledge to be extensive, although not necessarily integrated, due to the imminent confirmation of their baccalaureate degree. Thus, one would expect their performance on Task I to reflect their knowledge and understanding of functions. For this reason their performance on Task I in general and certain specific items is particularly troubling. The evidence suggests that these errors represent misconceptions about the definition of function, function notation and evaluation, composition of functions, and particularly, functions presented graphically. As suggested earlier, if teachers hold a limited and impoverished understanding of functions, they may have few tools with which to respond to genuine student questions and dilemmas. Their responses on Task II will provide further evidence of their subject-matter knowledge and confirm or refute these findings.

Beliefs: Beliefs Scale

The beliefs of both groups were very similar as measured on the 32-item Beliefs Scale with internal consistency within groups of questions. Statistical analysis using the t-test of differences in means indicated that the two groups differed significantly at the .01 level on only one of the 32-items. It was interesting to note that the "lab" students were inclined to agree that "instruction can best remediate poor computational performance by the deliberate teaching of correct rules" whereas the undergraduates clearly disagreed with this statement. Given that the Beliefs Scale was administered during the "lab" students initial experience in the program and that this statement reflects a long-held, pervasive view of "best practice", their agreement is not particularly surprising. The lack of agreement by the undergraduates can be attributed to their recent experiences in general education courses which undoubtedly emphasize conceptual understanding. The following table, Table 4, provides a list of strongly held beliefs in each of the categories.

Table 4. Beliefs Scale Summary of Areas of Agreement

1. Beliefs About the Learner

- Agree with the constructivist's view of student learning.
- Agree with the role of the teacher as facilitator.
- Strongly agree that "all students can learn mathematics" and that no "special" abilities are necessary.

2. Beliefs About Learning Mathematics

- Strongly agree that "modeling mathematical ideas through the use of representations (concrete, visual, graphical, symbolic) is central to the teaching of mathematics."
- Disagree that ideas should be explained in terms of formulas.
- Strongly agree that learning should be embedded in authentic problem situations.
- Strongly agree that learning should promote the development of both concepts and procedures.

3. Beliefs About Mathematics

- Strongly agree that applications and modeling of data reflect mathematics.
- Strongly agree that reasoning and logic represents mathematics.

Thus, these findings suggest that the task of analysis is not one of group comparison but of explaining the results. The concept of a belief system is a metaphor for examining and describing how an individual's beliefs are organized (Green, 1971; Rokeach, 1960). In the same way that researchers think of a cognitive structure in a particular concept domain, they have also come to consider a belief system; that is, a belief system is dynamic in nature and subject to change and restructuring as individuals consider and evaluate their beliefs in light of their experiences. Several theories of belief systems have emerged and provide a theoretical framework for research on individuals' beliefs.

Green (1971) identified three dimensions of belief systems that address how the beliefs are related to each other. The first of these dimensions is concerned with the dependence of beliefs on each other; that is, that beliefs are seldom held in isolation or total independence but related to each other logically as reasons are related to conclusions. In Green's systems some beliefs are *primary* while others are *derivative*. For example, a teacher who holds the belief that it is important to present mathematics "clearly" to the students might also believe that it is important to structure a specific sequence and order of examples. While the belief that the mathematics should be presented "clearly" would be primary, the beliefs concerning the specificity and order of the sequence would be derivative. Green's second dimension is related to the degree of conviction or psychological strength of these beliefs. Strongly held beliefs are considered *central* while *peripheral* beliefs are considered the most susceptible to examination and/or change. Green argues that "logical primacy" and "psychological centrality" are orthogonal dimensions; that is, they are two different features or aspects of a belief. The third of Green's dimensions has to do with the claim that "beliefs are held in clusters and protected from any relationship with other sets of beliefs" (Green, 1971, p.48). He suggests that this clustering of beliefs prevents "cross fertilization among clusters of beliefs or confrontations between them" (p.48) and thus, makes it impossible to hold conflicting sets of beliefs. Green's theory in many ways helps to explain the inconsistencies among beliefs professed by teachers, documented in several studies (Brown, 1985; Cooney, 1985; Thompson, 1982,1984). Thus, according to both Green and Rokeach, it is possible for a teacher to hold beliefs simultaneously that doing mathematics is synonymous with problem solving and that the best way to learn mathematics is by breaking it down into a sequence of skills to be mastered one at a time.

For Rokeach (1960) belief systems exist along a continuum of openness/closedness. A closed system would be one in which things are either right or wrong, black or white, and shades of gray do not exist. Perry (1970) proposes a system of intellectual development that focuses on the individual's relationship to authority. A person who believes that an authority figure is all-knowing also tends to believe that answer to questions are based on that authority and are thus, a singular response. Perry characterizes this as dualistic thinking which is quite similar to Rokeach's closed belief system. This notion of how people orient themselves to authority is the common thread that runs through Rokeach's, Perry's, and Green's analysis. Evidentially held beliefs are those that are held with regard to evidence. Thus, they are beliefs which may change in light of further evidence. Non-evidentially held beliefs cannot be changed by the introduction of evidence. These are the beliefs which when challenged prompt the response, "Don't bother me with the facts, I have made up my mind" (Green, 1971, p.48).

The results of the administration of the Beliefs Scale in Table 4 represent strongly held and thus, psychologically central beliefs. They would also seem to represent primary beliefs rather than derivative beliefs. With respect to Beliefs about the Learner, the results indicate a consistent view that learning occurs when children construct knowledge, teachers facilitate this process, and all children are capable of learning mathematics.

Derivative beliefs would take the form of how this kind of learning might take place. Those beliefs are reflected in the responses to the category, Beliefs about Learning Mathematics, in which the emphasis on learning that is embedded in authentic problem situations and which promotes the development of both concepts and procedures is consistent with the roles of students and teachers in the first category. The Beliefs about Mathematics espoused in the third category, that mathematical activities are comprised of applications which are modeled from real data, reasoning and logic, would also seem to be primary rather than derivative. The question of whether the beliefs are evidentially or non-evidentially held cannot be determined directly from the survey.

The results for both groups seem to represent a consistent belief system. However, since inconsistencies between professed beliefs and instructional practices such as those reported by McGalliard (1983) do exist, they should "alert us to important methodological considerations; that is, any serious attempt to characterize a teachers' conceptions of the discipline should not be limited to an analysis of the teachers' professed views - it should also include an examination of the instructional setting, the practice characteristic of that teacher, the relationship between the teacher's professed views and actual practice" (Thompson, 1992, p.34). Particularly with pre-service teachers, she advocates the collection of "data about their mathematical behavior as they encounter tasks in training courses" (p.134). This information, she suggests would be valuable to reform efforts in mathematics teacher education. Thus, in Task II, the prospective teachers' instructional practices as they respond to genuine student questions, naive conceptions, and dilemmas are considered.

Instructional Practices: Task II

Recall that Task II was analyzed along three dimensions - **teacher level of discourse/communication**, **student level of discourse/communication**, and **notion of mathematical authority**. Table 2 provides the descriptions of each level within the three dimensions. For example, on Vignette A, ten (5 "lab" students and 5 undergraduates) responses to the students' comment were scored as (2,1,1). This means that their response was **active (2)** along the level of teacher discourse, **passive (1)** along the level of student discourse, and **enactive (1)** along the level of mathematical authority. Thus, these prospective teachers suggested responses that provided a correct explanation that may have noted the distinction between the path of the ball and the graph of distance vs. time with an example that was related to the real-world context of the situation.. The results of the analysis of Task II are reported in Table 5 which follows. All of the vignettes were read and analyzed by both researchers as well as a trained research assistant to achieve an inter-rater reliability of .89.

Table 5. Dimensional Analysis of Classroom Interactions

Vignette Distributions	A Translation of Physical Path to Graph	B Translation of Graph to Physical Interpretation	C Function Evaluation	D Function Composition	E Inverse Functions
Undergrads	(2,1,1) - 5 (2,1,2) - 4 (2,3,2) - 1	(2,1,3) - 6 (2,1,2) - 1 (2,2,2) - 1 (2,2,3) - 1 (3,3,2) - 1	(2,1,3) - 5 (2,2,3) - 3 (2,2,2) - 1 (3,3,2) - 1	(2,1,2) - 1 (2,1,3) - 2 (2,2,3) - 2 (2,3,2) - 1 (2,3,3) - 3 (3,3,2) - 1	(2,1,2) - 2 (2,1,3) - 3 (2,2,2) - 2 (2,2,3) - 1 (3,2,2) - 1 (3,3,2) - 1
"Lab" Students	(2,1,1) - 5 (2,1,3) - 2 (1,2,1) - 1 (3,3,1) - 1 (3,3,2) - 1	(2,1,3) - 6 (2,2,3) - 3 (3,3,1) - 1	(2,1,3) - 8 (2,2,3) - 2	(2,1,2) - 1 (2,1,3) - 6 (2,2,3) - 2 (3,3,3) - 1	(2,1,3) - 4 (2,2,3) - 5 (3,3,3) - 1

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To examine the most frequently occurring response, we turn to the Table 6,

Table 6. Most Frequently Occurring Triads (in order of frequency)

Triad	Number of Undergrads	Number of Lab Students	Total Responses
(2, 1, 3)	16	26	42
(2, 2, 3)	7	12	19
(2, 1, 1)	5	5	10
(2, 1, 2)	8	1	9
(3, 3, 2)	4	1	5
(2, 2, 2)	4	0	4
(2, 3, 3)	3	0	3
(3, 3, 1)	0	3	3
(2, 3, 2)	2	0	2
(3, 3, 3)	0	2	2
(3, 2, 2)	1	0	1

The most frequently occurring response (42%) is clearly teacher directed with the teacher providing an explanation or sharing knowledge, the student receiving knowledge, and mathematical authority established through a review or establishment of formal definitions, symbolic representations, and algorithms or procedures. In some cases (23%), students are asked to explain their procedure or logic to the class for feedback. In an equal number of cases, the teacher utilizes either real-world contextual situations(10%) or perhaps numerical substitutions, technological tools, counter-examples or contradictions (9%) to establish mathematical authority while the students remain passive listeners. Approximately 16% of the responses could be classified as truly investigative in which the teacher orchestrates or suggests alternative pathways through a genuine invitation for student to engage in mathematical inquiry or explain their reasoning to the class.

In terms of between-group differences, the responses do not differ significantly. Both the undergraduates and the "lab" students have approximately the same number of teacher directed versus investigative responses. With respect to the quality of the responses, the undergraduates generated some very creative responses to genuine student dilemmas. In Task A, they confirmed the observations, but also noted the distinction between distance vs. time and distance vs. distance. In Task D and E, where conflicts were suggested, they were far more likely to suggest that students explore the mathematics and resolve the conflict through their own efforts. In several cases, this resolution utilized technology to provide an alternative source of mathematical authority. One of the responses to Task C requested students to consider the relationship between the graphs of f(x) and f(x+1) to determine which of the graphs of the algebraic solutions are correct and to utilize the TI-81 to graph the four choices. However, the undergraduates were also much more likely to exhibit misconceptions with respect to projectile motion and in diagnosing students' errors on Task C.

The responses of the "lab" students on the vignettes were fairly traditional teacher directed. In terms of the quality, the "lab" students exhibited very few misconceptions on any of the tasks. In particular, the real-world tasks, A and B were very strong and extended more than any others to include other concepts and illustrate the connections with real world phenomena. Throughout the tasks, the "lab" students consistently utilized accurate scientific information in their explanations.

Discussion and Conclusions

Within the research on teaching and teachers, they key questions have moved slowly along a continuum from "what they were" to "what they did" to "what they decided" to finally "what they believed.". Not only has the unit of investigation shifted significantly but so also has the orientation and expectations with respect to the outcomes from research on teachers and teaching. The means of describing these outcomes have moved from deterministic methods toward more descriptive ones.

These descriptions and the research findings they represent (many coincident with Shulman's initial description of pedagogical content knowledge) have provided us with wonderful stories about life in the classroom and about why teachers behave as they do. Certainly these descriptions can better enable us to understand and focus on what is truly important. However, it is also essential to consider for what purpose these descriptions are intended. If they simply provide an alternative form of empirical density that obscures the issues, then they are not useful. If they simply provide further evidence of teachers' deficiencies they are also not useful.

In considering research on teachers and teaching, it is essential to recognize the associated complexities. Each teacher represents a dynamic system of knowledge, beliefs, and instructional practices. Seeking to alter or change any aspect of their knowledge and beliefs will obviously have an impact on the system and influence their instructional practices - the question, of course, is how. Too frequently, the response to research findings, that suggest a certain group of teachers will lack the mathematical sophistication necessary to promote the kind of reform being called for by the National Council of Teachers of Mathematics, is to provide them with *more* of what they seem to lack - mathematics, pedagogy, curricular materials, etc. For example, a solution to the subject-matter difficulties of the undergraduates, as revealed in Task I, might be to take more mathematics courses. However, even when the "lab" students demonstrated understanding of a particular topic such as evaluation of a function presented graphically (question #14 - 4 undergrads answered incorrectly, 0 "lab" students answered incorrectly), that understanding did not translate into features of a vignette response that considered students' conceptions or their ability to make sense of the mathematics for themselves. The difficulty of this approach is that is fails to consider the inherent complexity of the system that characterizes each teachers' knowledge, beliefs, and instructional practices.

The analysis of the data for the two groups suggests that comparisons at different points in their respective programs is problematic. With respect to the assessment of subject-matter knowledge (Task I) and beliefs (Beliefs Scale), the two groups were very similar and it was reasonable to administer the assessments when we did. The profile of the beliefs, especially of the "lab" students, suggests that these primary, psychologically central beliefs would focus on all children constructing their own mathematical knowledge with teacher facilitation. The fact that the "lab" students expressed these beliefs at the beginning of their program is significant and bodes well for the development of their pedagogical content knowledge. However, although their subject-matter knowledge was strong and their expressed beliefs were consistent with a constructivist view of learning, they were unable to suggest responses to students that capitalized on their knowledge and beliefs. One possible explanation is that at the beginning of their program, they only had their own learning experiences to draw upon.

The results of Task II for both groups are consistent with the findings of Even and Markovits (in press) - the majority of the prospective teachers' responses to vignettes of students' misconceptions were also teacher centered/directed indicating a transfer of knowledge by direct telling, and few were rich in conceptual meaning. For the "lab" students with their limited experiences, these findings are not unexpected. For the undergraduates in their capstone course, these findings, although consistent with previous research, are problematic. However, what is evident from the few strong responses is the lack of what we have traditionally thought of as explaining in mathematics - that is telling or presenting. It was also the case that those undergraduates exhibited the strongest subject-matter knowledge. Thus, the connection between strong subject-matter knowledge and the willingness to abandon the traditional view of teaching mathematics as presenting information, is certainly present for those prospective teachers.

Their beliefs about learners and learning mathematics also played an important role in determining the nature of the responses. The beliefs that students can and do learn mathematics through their own mathematical inquiry is essential to assuming the role of facilitator. It was also essential that these prospective teachers believed that mathematics makes sense - that it is much more than a collection of rules and procedures. These beliefs about mathematics provided the teachers with the foundation to engage in mathematical inquiry and to value the importance of students forming and testing conjectures. The pedagogical tools of utilizing student-to-student interactions and technology as a source of authority were also important components of these explanations. Other means of engaging in mathematics were necessary to replace the traditional methods of lecture/discussion. Thus, the strong explanations exhibited by these few undergraduates were dependent on all of the sources of pedagogical content knowledge - strong subject-matter knowledge, student-centered beliefs, and knowledge of students' understanding.

Implications for Teacher Education

One of the contributions of this study within the current research on teachers is that it carefully considered the complexities associated with pedagogical content knowledge for two important groups of prospective teachers. Rather than focusing on one aspect of knowledge and/or beliefs that contribute to pedagogical content knowledge and inferring the nature of teachers' instructional practices, this study considered several aspects of knowledge and beliefs and relationship to specific instructional practices. This study also contributes to the research concerning prospective teachers and suggest potential programmatic implications. The use of the vignettes task in which the teachers first considered students' conceptions and developed explanations or responses provided a valuable activity for teachers. By asking them to consider students' conceptions prior to planning or teaching lessons, the emphasis is shifted from a *presentation* mode in which the mathematics is the focus to an *interactive* mode in which the students' construction of knowledge is the focus. While not all prospective teachers will focus on students' construction of knowledge, few will find this focus without specific attention to students' conception is their methods classes.

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APPENDIX A

Beliefs Scale

On the following pages are a series of statements. There are no correct answers for these statements. They have been set up in a way which permits you to indicate the extent to which you agree or disagree with the ideas expressed. Suppose the statement is:

1. I enjoy teaching mathematics.

As you read the statement, you will know whether you agree or disagree. If you strongly agree, shade-in circle A opposite number 1 on the answer sheet. If you agree but with reservations, that is you do not fully agree, shade-in the circle B. If you disagree with the idea, indicate the extent of your disagreement by shading-in circle D for disagree or circle E for strongly disagree. If, however, you neither agree nor disagree, that is you are not certain, shade-in circle C for undecided. Now mark your answer sheet. Be sure to answer every question. Work quickly but carefully. There are no "right" or "wrong" answers. The only correct responses are those that are true _for you_. Respond to each question using this guide.

Beliefs Scale (continued)

A - strongly agree

B - agree

C - undecided

D - disagree

E - strongly disagree

1. Students construct meaning as they learn mathematics.
2. Students use what they are taught to modify their prior beliefs and behavior.
3. All students can learn mathematics.
4. The role of the teacher in a secondary mathematics classroom is to facilitate learning.
5. Students learn by remembering what they are taught.
6. Students learn by recording and storing information.
7. Learning mathematics requires special ability which most students do not have.
8. The role of the teacher in a secondary mathematics classroom is to transmit knowledge.
9. Modeling mathematical ideas through the use of representations (concrete, visual, graphical, symbolic) is central to the teaching of mathematics.
10. Representations serve as vehicles for examining mathematical laws.

Beliefs Scale (continued)

11. The best way to model mathematical ideas is to decompose them into a sequence of basic skills which can be mastered one at a time.
12. Mathematical ideas should be examined in terms of a basic formula or standard equation.
13. Instruction should build on students' prior knowledge.
14. Learning situations should be embedded in authentic problem situations that have meaning for the students.
15. Instruction should make the mathematical structure explicit so that the students can build on that structure.
16. Learning situations should be based on an analysis of mathematical structure.
17. Instruction should promote the development of both conceptual and procedural knowledge.
18. Instruction can best remediate poor computational performance by the deliberate teaching of correct rules.
19. It is important to determine whether conceptual or procedural knowledge contributes more to mathematical expertise.

Beliefs Scale (continued)

20. Students must first construct meaning for symbols so that they can develop procedures for manipulating symbols based on the meanings of the symbols.
21. Mathematical problem solving is nearly synonymous with doing mathematics.
22. Problems and applications provide an excellent means to introduce new mathematical content.
23. Doing mathematics involves mastering a set of rules and procedures that may be applied to solve problems.
24. Students should gain practice manipulating expressions and practicing algorithms as a precursor to solving problems.
25. An important component of mathematical thinking is the process of mathematical modeling.
26. Learning situations should involve the collection and analysis of real data, construction of a graph or visual display of the data, and the construction of an equation that most closely models the data.
27. Mathematical thinking should be nurtured through exposure to classical mathematical applications such as projectile motion and exponential growth.
28. Realistic mathematical situations contain too much "messy data" for students to handle.

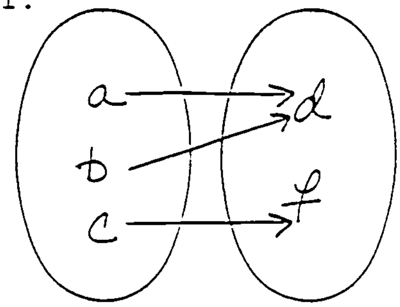
Beliefs Scale (continued)

29. An important component of mathematical thinking is the process of forming hypotheses or conjectures and establishing their validity both intuitively and more formally.
30. Classroom discourse should be structured to promote mathematical argument.
31. Instruction should provide the opportunity for students to engage in the construction of both inductive and deductive proofs.
32. Classroom discourse should be structured so that students experience the opportunity to present their proofs to the rest of the class.

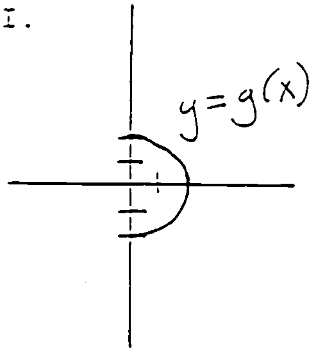
Assessment of Subject-Matter Knowledge Concerning Functions and Graphs

1. Which of the following relations displayed below describe a function?

I.



II.



III.

x	f(x)
3	5
6	2
3	7
7	-1

a. I, II, III

b) I, III

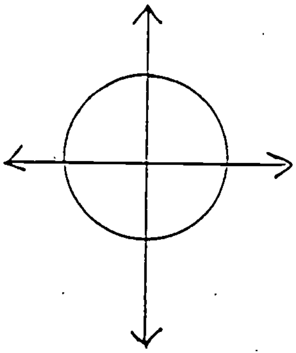
e) I only

c) I, II

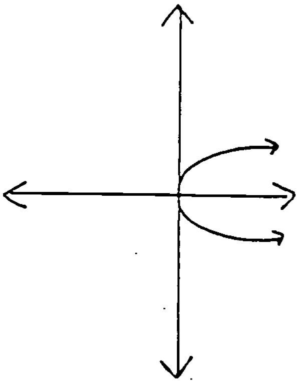
d) II, III

2. Determine which of the following graphs represents a function?

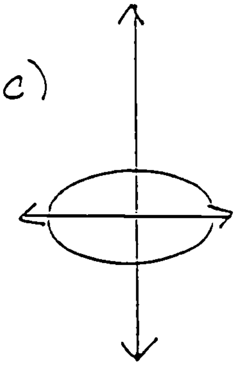
a)



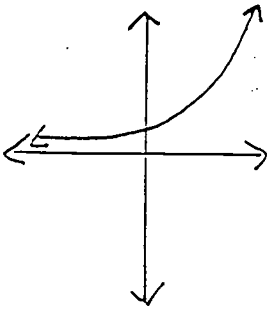
b)



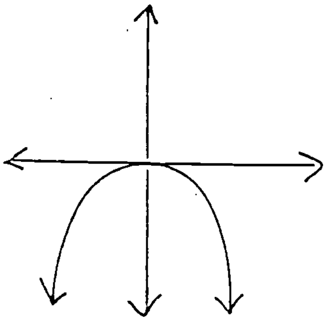
c)



d)



e)



3. Determine the domain of the function $f(x) = \sqrt{2x+3}$.

a. $[-1,\infty]$

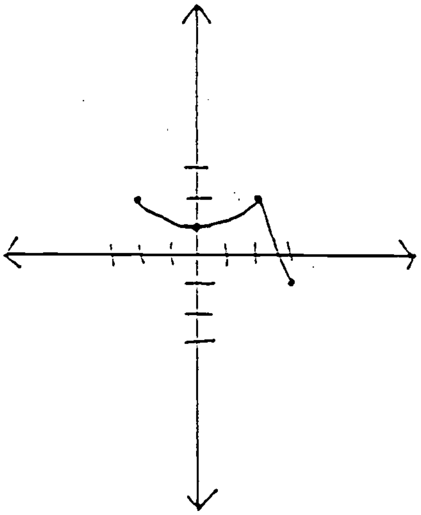
b) $[-3/2, \infty]$

c) $[0, \infty]$

d) all real numbers

e) none of the proceeding

4. The graph of a function f(x) is given below. Which of the following statements is/are true?



I. $f(f(3.5)) > 0$

II. f(x) is a one-to-one function

III. The range of f(x) is [-1,2]

a) I is the only true statement

b) II is the only true statement

c) III is the only true statement

d) I, II are the only true statements

e) I, III are the only true statements

Assessment of Subject-Matter Knowledge (continued)

5\. Let $f(x) = \begin{cases} 2x & \text{if } x < -2 \\ 4x^2 & \text{if } -2 \leq x \leq 2 \\ 1 & \text{if } x > 2 \end{cases}$ determine $f(5)$.

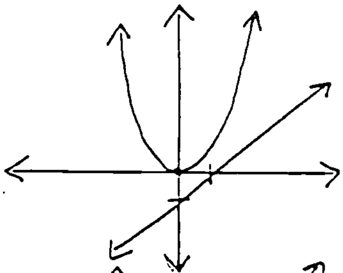
a) 5 b) 10

c) 1 d) 100

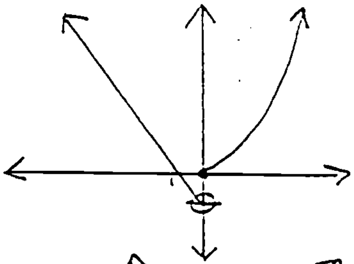
e) none of the above

6\. If $f(x) = \begin{cases} x^2 & \text{if } x \leq 0 \\ x-1 & \text{if } x > 0 \end{cases}$ determine which of the following graphs best represent $f(x)$.

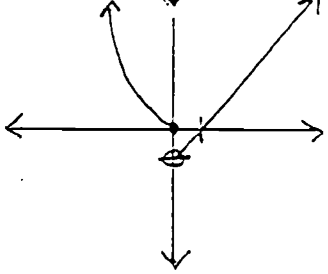
a)



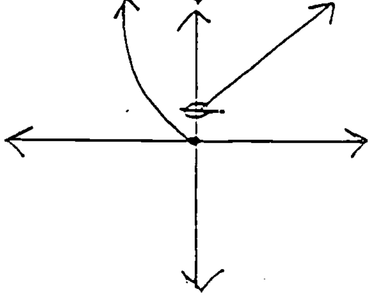
b)



c)



d)



e) none of the preceding

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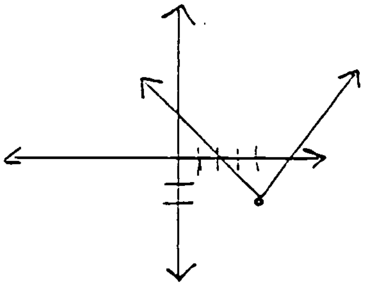
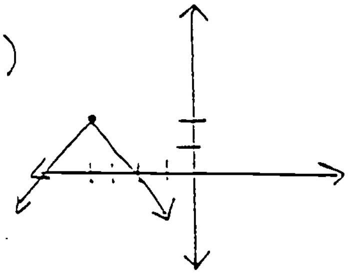
Assessment of Subject-Matter Knowledge (continued)

7. Which of the following graphs best represent the graph of the function, $f(x) = 2 - |x-4|$?

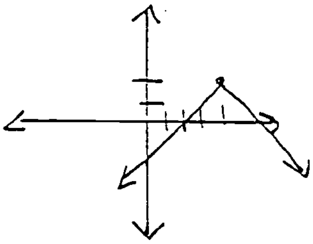
a)



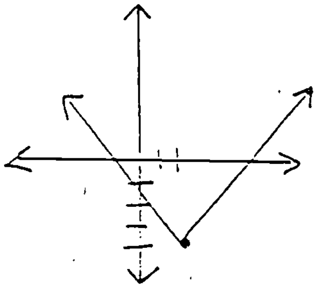
b)



c)



d)



e. none of the preceding

8. Let $h(x) = 2x^3$. Determine $f(x)$ and $g(x)$ such that $h(x) = f[g(x)]$.

a) $f(x) = x^3$, $g(x) = 2x$

b) $f(x) = 2x^2$, $g(x) = x$

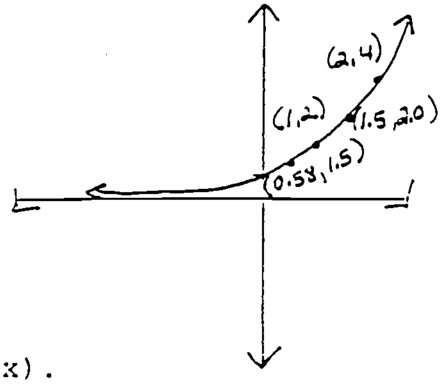
c) $f(x) = 2$, $g(x) = x^3$

d) $f(x) = 2x$, $g(x) = x^3$

e) none of the preceding

Assessment of Subject-Matter Knowledge (continued)

9. Use the accompanying graph of $f(x)$ to evaluate $f^{-1}(1.5)$.



a) 0.58 b) 2.8

c) 2.25 d) $2\sqrt{2}$

e) none of the preceding

10. Let $f(x) = 2^{x-3}$. Determine $f^{-1}(x)$.

a) $f^{-1}(x) = (\frac{1}{2})^{x-3}$ b) $f^{-1}(x) = \log_2(x+3)$

c) $f^{-1}(x) = \log_2(x-3)$ d) $f^{-1}(x) = \log_2(x)+3$

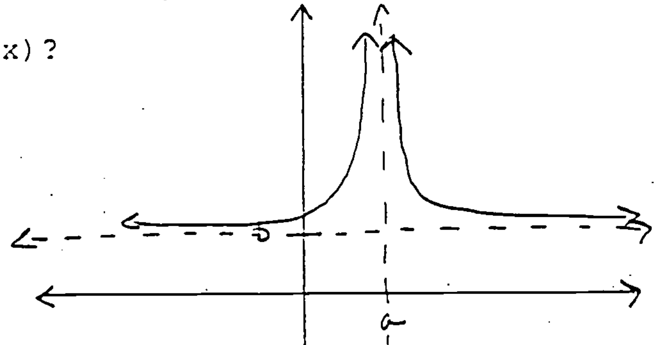
e) none of the preceding

11. Let $f(x) = 2x-3$, and $g(x) = x^2+4x-1$. Classify $(g \circ f)(x)$ as linear, quadratic, cubic, or none of these.

a) linear b) quadratic

c) cubic d) none of these

12. Which of the following functions best describes the given graph of $f(x)$?



a) $f(x) = 1/(x-a)^3 + b$ c) $f(x) = (x-a)2+b$

c) $f(x) = 1/(x+a)^3 + b$ d) $f(x) = 1/(x-a)^2+b$

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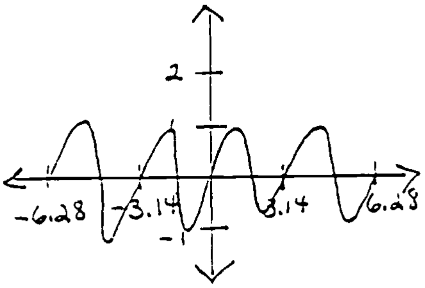
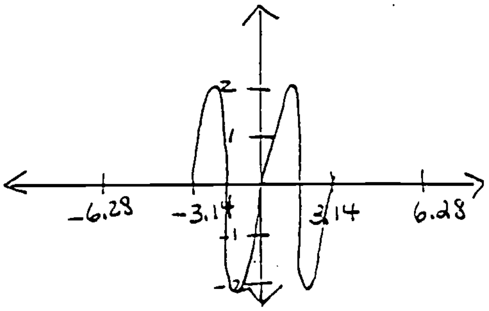
Assessment of Subject-Matter Knowledge (continued)

13. If $f(x) = 2x$ and $g(x) = \sin(x)$, which of the following represents the graph of $h(x) = f[g(x)]$?

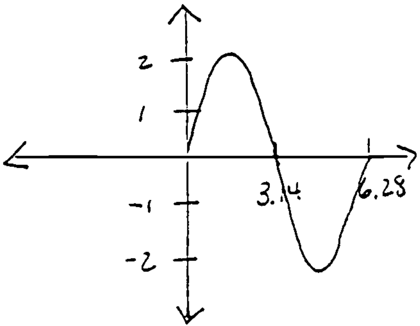
a)



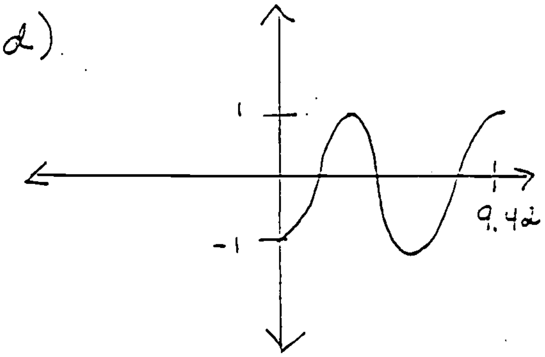
b)



c)



d)



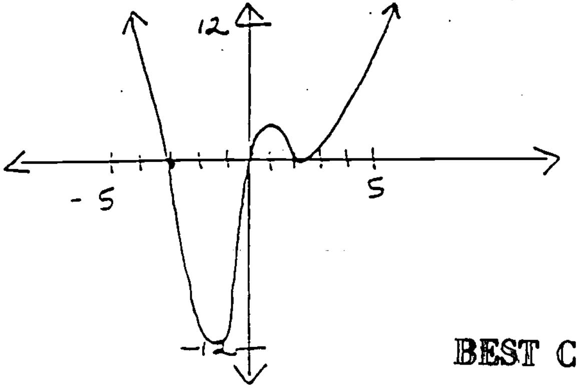
e) none of the preceding

14. A sketch of $f(x)$ is shown below. Which statement(s), if any, about the function is/are true?

I. $f(0) > f(-2)$ II. $f(-3) < 0$

a) I & II are true b) I only is true

c) Neither I nor II are true d) II is true



Assessment of Subject-Matter Knowledge (continued)

15. Suppose the function f(x) is quadratic. Determine which, if any of the following functions are also quadratic?

I. $f(x-\frac{1}{2})$ II. $f(x)$ III. $1-2f(x)$

a) Only I and II are quadratic b) All three are quad.

c) Only II & III are quadratic d) I & III are quad.

Vignettes Task

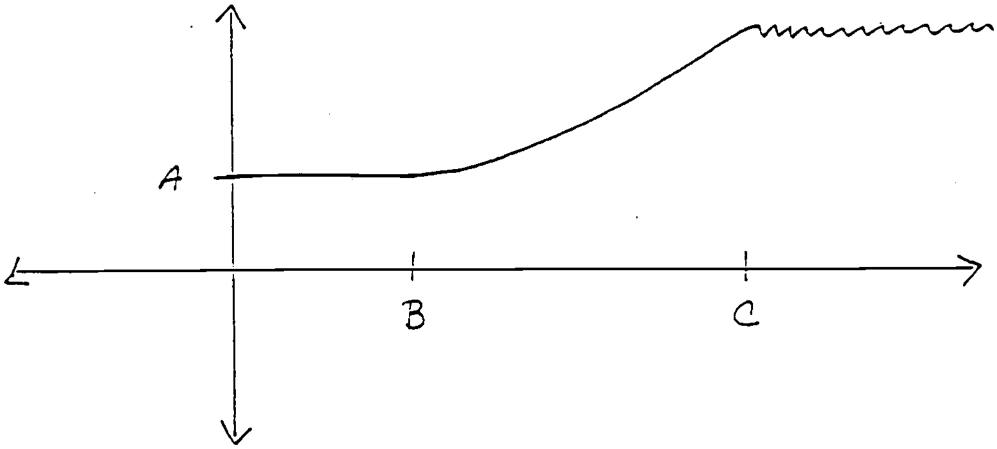
Task A

Suppose that in teaching a unit on functions and graphs you are explaining a typical projectile motion problem in which a ball is tossed at t=0 seconds and hits the ground again at t=7 seconds. You sketch the graph of h(t) in order to explain the relationship between height and time in this scenario. A student says, " Wow, the graph looks just like the ball going up and coming back down!"

How do you respond to this students' comments?

Task B

Suppose that you are discussing the definition of function in class. You illustrate the concept with a qualitative graph such as the one below and pose the following questions.



1. How would you describe the temperature in the following time interval? [0,B], [B,C], and [C, ∞]

2. What real-world situation could be described by this graph? Explain the temperature in the various time intervals in terms of your situation.

Task E

You have been discussing the concept of inverse functions in class. You pose the following problem in class.

Determine the inverse (f^{-1} of the function $f(x) = x/7 + 4$.

One student suggests that "$f^{-1}(x) = 7x-4$". Another student says, "No, I think it's $f^{-1}(x) = 7(x-4)$."

How would you respond to these comments?

Vignettes Task (continued)

Task C

Suppose that several students chose the following solutions to his problem involving evaluation of a function.

Problem: Evaluate $f(x+1)$ if $f(x) = x^2+x+1$.

a) x^2+3x+2 b) x^2+x+2

c) x^2+x+3 d) x^2+3x+3

For each of the incorrect solutions:

1. What is the source of each mistake? (Show how you may have found this solution.
2. How would you respond to these incorrect solutions?

Task D

You have been discussing the concept of composition of functions in class. You pose the following problem in class.

Let $h(x) = f[g(x)]$ and determine $f(x)$ and $g(x)$ if $h(x) = 2(x-5)^2$

One student suggests that "$g(x) = (x-5)2$ and $f(x) = 2$". Another student interrupts, "No, $f(x)$ must be equal to $2x$ if $g(x) = (x-5)^2$." A third student remarks, "Well I think $g(x) = (x-5)$ and $f(x) = 2x^2$." The class seems confused.

How would you respond to these comments and clear up the confusion?

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